Probing Gravity with Spacetime Sirens

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ABSTRACT

A gravitational observatory such as *LISA* will detect coalescing pairs of massive black holes, accurately measure their luminosity distance and help identify a host galaxy or an electromagnetic counterpart. If dark energy is a manifestation of modified gravity on large scales, gravitational waves from cosmologically-distant spacetime sirens are direct probes of this new physics. For example, a gravitational Hubble diagram based on black hole pair luminosity distances and host galaxy redshifts could reveal a large distance extra-dimensional leakage of gravity. Various additional signatures may be expected in a gravitational signal propagated over cosmological scales.

1. Introduction

Evidence for an accelerating expansion of the Universe is getting stronger. A measurable dimming of distant type Ia supernovae (e.g., Astier et al. 2006; Riess et al. 2007; Wood-Vasey et al. 2007), CMB data (e.g., Spergel et al. 2007) and additional cosmological probes (e.g., Wright 2007) indicate that a repulsive "dark energy" comprises 73% of the Universe's content. Characterizing the properties of dark energy and uncovering its physical nature are two of the most important goals of modern cosmology. Current observational strategies include tests of the possibility that dark energy arises from a failure of general relativity on cosmological scales (Albrecht et al. 2006).

Essentially all astronomical measurements are performed via electromagnetic waves. The availability of accurate gravitational wave measurements, within the next decade or so, will thus be a significant development. In particular, since the propagation of photons and gravitons could differ at a fundamental level, we argue here that gravitational waves emitted by cosmologically-distant "space-time sirens," such as coalescing pairs of massive black holes, may be used as valuable alternative probes of dark energy physics.

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Black holes with masses $\gtrsim 10^6 M_\odot$ are present at the center of numerous nearby galaxies (e.g. Kormendy & Richstone 1995; Magorrian et al. 1998). As such galaxies collide over cosmic times, their central black holes coalesce, releasing $\gtrsim 10^{58}$ ergs of binding energy in the form of gravitational waves (hereafter GWs). To measure the GWs emitted by these cosmologically-distant space-time sirens, ESA and NASA will build the Laser Interferometer Space Antenna, LISA[1].

2. Gravitational Measurements

GWs emitted by black hole binaries have the unusual property of providing a direct measure of the luminosity distance, D_L, to the black holes, without extrinsic calibration. Owing to the highly coherent nature of GW emission (Schulz 1986), the amplitude (or strain), $h_{+\times}$, frequency, f, and frequency derivative, $\dot{f}$, of the leading order (quadrupolar) GW inspiral signal scale as

$$h_{+\times}(t) \propto \frac{\left[(1+z)M_c\right]^{5/3} f^{2/3}}{D_L}, \tag{1}$$

$$\dot{f}(t) \propto \left[(1+z)M_c\right]^{5/3} f^{11/3}, \tag{2}$$

where $+\times$ represents the two transverse GW polarizations, $M_c = (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$ is the black hole pair "chirp" mass and z its redshift. Provided the GW source can be reasonably well localized on the sky, an extended observation of the chirping signal leads to precise measurements of $h_{+\times}$, f, $\dot{f}$ and thus D_L, independently. LISA's orbital configuration allows for a "triangulation" of GW sources on the sky to within a solid angle $\delta\Omega \sim 1$ deg^2 (Cutler 1998; Vecchio 2004), thus providing very accurate distance measurements, with $\delta D_L/D_L < 1\%$ at $z \lesssim 2$ typically (Cutler 1998; Hughes 2002; Vecchio 2004; Lang & Hughes 2006).

Recently, the possibility of identifying the individual host galaxy in which a pair of merging black holes seen by LISA is to be found has been explored in some detail (Holz & Hughes 2005; Kocsis et al. 2006, 2007). Prospects for such identifications at redshifts $z \lesssim 3$ are good and identifications out to $z \sim$ 5-7 may even be possible in some cases (Kocsis et al. 2007). A unique host galaxy identification can be achieved through coordinated observations with traditional telescopes, either to survey the LISA-triangulated area for unusual galactic properties or central activity after the coalescence, or by monitoring in real time the sky area for

[1]`http://lisa.nasa.gov/`

unusual electromagnetic emission, as the coalescence proceeds. Typically $\gtrsim 10^{53}$ ergs of kinetic energy are delivered to the recoiling black hole remnant (e.g., Schnittman & Buonanno 2007, and references therein) and its environment. The disturbed gas surrounding coalescing black holes could thus power bright electromagnetic emission during and after coalescence (Armitage & Natarajan 2002; Milosavljevic & Phinney 2005; Dotti et al. 2006), permitting the coincident identification of a unique host galaxy.

3. Gravitational Hubble Diagram

A consequence of successfully identifying the host galaxies of coalescing black hole pairs is the possibility to draw a gravitational Hubble diagram, i.e. one that relates the gravitational luminosity distances, D_L, of space-time sirens to the electromagnetic redshifts, z, of their host galaxies.

The solid line in Fig. 1 shows a model fit to a Hubble diagram from a recent type Ia supernova sample with a dark energy equation of state $P/\rho \equiv w = -1.05 \pm 0.3$ (at 3σ, shown by the shaded region). Luminosity distances are expressed in terms of the usual distance modulus, $\mu = 5 \log_{10}(D_L/\,1\,\mathrm{Mpc}) + 25$. The individual supernova data points of the gold sample of Riess et al. (2007) are also shown. A flat universe with $H_0 = 72$ km s^{-1} Mpc^{-1} and a matter density $\Omega_m = 0.27$ is assumed.

The general interest of a gravitational Hubble diagram can be illustrated by considering alternative explanations for the apparent dimming of distant type Ia supernovae. If this dimming were caused by dust attenuation, photon-axion conversions or an intrinsic evolution of the supernova population (e.g., Csáki et al. 2002; Mirizzi et al. 2006; Robaina & Cepa 2007), rather than stretched distances, this would become apparent in comparisons between gravitational and electromagnetic Hubble diagrams. Indeed, in each of these alternative dimming scenarios, photon- and graviton-based Hubble diagrams would be fundamentally discrepant since self-calibrated gravitational distance measurements are not susceptible to any significant bias from absorption, scattering, reddening, or axion-conversion.

In practice, however, the value of such a comparison is limited by line-of-sight matter inhomogeneities, which generate "weak lensing" uncertainties on any photon or graviton D_L measurement (Holz & Hughes 2005; Kocsis et al. 2006; Dalal et al. 2006). While the lensing effect can be averaged out over the many random lines-of-sights available with the supernova data (i.e., data points in Fig. 1), it may not be the case for coalescing black hole pairs if LISA merger event rates are modest (e.g., a few tens per year at $z \lesssim 5$; Menou et al. 2001; Wyithe & Loeb 2003; Sesana et al. 2004; Micic et al. 2007). Weak lensing errors on

individual measurements amount to distance uncertainties ranging from $\delta D_L/D_L \simeq 1\%$ at $z = 0.5$ to $\delta D_L/D_L \simeq 10\%$ at $z = 5$ (e.g. Kocsis et al. 2006). The corresponding 3σ distance modulus uncertainties exceed the 3σ confidence contours on the current dark energy model fit (shaded region in Fig. 1) at $z \gtrsim 0.7$, which makes comparisons between photon- and graviton-based Hubble diagrams imprecise even at moderate redshifts. The extent to which LISA events can be used to perform meaningful comparisons with photon-based Hubble diagrams will thus depend strongly on the actual distribution of massive black hole merger events with redshifts and the corresponding efficiency of host galaxy identifications.

4. Modified Gravity

The greatest prospect for dark energy science with gravitational waves may lie in exploring new physics on cosmological scales. The possibility that accelerated expansion results from a failure of general relativity has fueled much theoretical work on large scale modifications of gravity over the past few years. Since building a satisfactory theory of modified relativistic gravity is a formidable task, any insight that can be gained from direct observational constraints on the linearized gravitational wave regime cannot be overlooked.

One may expect gravity modifications to contain a new length scale, let us call it R_c, beyond which gravity deviates from general relativity. In order to explain the observed accelerated expansion of the Universe, this scale is expected to be of the order of the current Hubble radius H_0^{-1}. Modified gravity must also pass standard tests of general relativity on scales much shorter than R_c, e.g. in the solar system and in the strong field regime of binary pulsars. An existence proof of modifications of this type is given by DGP gravity (Dvali et al. 2000), a braneworld model with an infinite (possibly many) extra dimension. In this model, which also leads to an accelerated expansion of the Universe (Deffayet 2001; Deffayet et al. 2002a), gravity is intrinsically higher dimensional[2]. To leading order, the gravitational potential has the standard $1/D$ behavior at distances D smaller than R_c, while it behaves 5-dimensionally at larger distances ($1/D^2$). Moreover, the stronger the gravitational field, the closer the theory is to general relativity. As a consequence the model passes the PPN tests in the solar system, with possible deviations emerging in upcoming generations of solar system measurements (Lue & Starkman 2003; Dvali et al. 2003). The GW emission of classical astrophysical sources is also expected to closely match that of general relativity. It is only at very large distances and low curvature that gravity is beginning to "leak" in the

[2]Gorbunov et al. (2006), Charmousis et al. (2006), Deffayet et al. (2006), Dvali (2006) Izumi et al. (2007) and Gregory et al. (2007) discuss the stability of the DGP self-accelerating phase.

extra dimension.

A previously unexplored consequence of extra-dimensional leakage is that cosmologically-distant GW sources would appear dimmer than they truly are, from the loss of GW energy flux to the bulk. Inspired by this idea, we investigate here the consequences of this possible modification of gravity, borrowing from the DGP model the notion that strong field gravity (and hence GW emission) asymptotes to general relativity, while deviations appear at very large, typically cosmological distances, in the weak field regime. We will not deal here specifically with GWs in the DGP model, which is the subject of a separate study, rather, we illustrate more generally how a gravitational observatory such as LISA may reveal cosmological deviations in the weak field graviton propagator.

In the presence of large distance leakage (say at distances much larger than R_c), flux conservation over a source-centered hypersphere requires that the GW amplitude scales with distance D from the source as

$$h_{+\times} \propto D^{-(dim-2)/2}, \tag{3}$$

where dim is the total number of space-time dimensions accessible to gravity modes. Thus, for $dim \geq 5$, it deviates from the usual $h_{+\times}(D) \propto 1/D$ scaling. The scaling in Eq. (3) is consistent with explicit GW calculations in spacetimes with compact extra-dimensions (Cardoso et al. 2003; Barvinsky & Solodukhin 2003) and also applies to models where extra dimensions open up only at large distances (Gregory et al. 2000; Dvali et al. 2000, 2001).

The top three lines in Fig. 1 show gravitational Hubble diagrams, i.e. the expected locus of D_L and z measurements from black hole pairs and host galaxies, in three simple scenarios with $dim = 5$ and leakage beyond a scale R_c obeying Eq. (3). To allow arbitrary possibilities for the transition at the cross-over scale, we adopt $h_{+\times}(D_L) \propto (D_L[1 + (D_L/R_c)^{n/2}]^{1/n})^{-1}$, where n determines the transition steepness. In the three cases shown, cross-over scales of a few Hubble distances, $R_c =$ 1-4 $D_H \sim$ 2-9 Gpc were used and values $n = 1$ or 10 ("steep") for the transition steepness were adopted. Error bars show the magnitude of 1σ weak lensing uncertainties on D_L measurements due to line-of-sight matter inhomogeneities.

The electromagnetic Hubble diagrams of each scenario is assumed to mimic the cosmology of a $w = -1$ dark energy model (solid line), as expected if these scenarios produce an accelerated background cosmological FLRW space-time which affects the geodesic motion of photons in the standard way. As a result of the different propagation of GWs and large distance leakage of GW energy flux, however, measured GW amplitudes, $h_{+\times}$, are reduced and gravitational D_L values are overestimated (Eqs. [1]–[2]) for sources beyond the cross-over scale in these scenarios. The corresponding graviton Hubble diagrams would thus deviate from the electromagnetic version (solid line) since all photon-based measurements are standard, by construction. The discrepancy between the graviton-measured and photon-inferred

D_L values could be of order unity even at moderate redshifts. Black hole merger events and associated host galaxies may thus reveal the leakage of gravity with cross-over scales $R_c \sim$ a few Hubble distances.

Diagnostics based exclusively on electromagnetic measurements cannot reveal gravitational leakage directly, although they may do so indirectly (e.g., Lue & Starkman 2004; Wang et al. 2007; Zhang et al. 2007). Importantly, the leakage may be equally difficult to identify with only a GW signal. As is well known, all the mass combinations that determine the GW signal, e.g. in post-Newtonian expansions of the inspiral signal, are redshifted by a factor $1 + z$ and thus degenerate with redshift (e.g., Hughes 2002). Provided that general relativistic black holes and GW emission are accurately recovered on sub-cosmological scales in the modified gravity scenarios considered, leakage on cosmological scales would lead to a hidden bias on the redshift (overestimated) and black hole masses (underestimated), as inferred from adopting a background cosmology that ignores leakage altogether. Only if the redshift of the host galaxy were to become available would the discrepancy with the gravitational measurement become apparent, as illustrated in Fig. 1. Paradoxically, this could limit our abilities to identify the host galaxy of a merging black hole pair, by invalidating search strategies that rely on a (biased) value of the gravitational luminosity distance to define redshift cuts on potential host galaxies (Kocsis et al. 2006).

We note that evidence of GW leakage could also emerge in different contexts, e.g. from low redshift short gamma-ray bursts detected by a network of ground-based GW detectors (Dalal et al. 2006), or from an unexpectedly low amplitude of the present-day GW background relative to its CMB-normalized value (A. Buonanno, priv. communication).

Despite the strong GW leakage suggested by the above energetic arguments, one should be very cautious in evaluating the significance of the deviations shown in Fig. 1. Indeed, Eq. (3) says very little about the cross-over transition physics and other details of GW propagation on cosmological scales. In particular, even though the scaling given in Eq. (3) does apply to the DGP model, the "infrared transparency" effect (Dvali et al. 2001) can be shown to result in a considerable increase of the distance at which GW leakage is manifested in DGP, reaching scales much beyond R_c for sources with frequencies relevant to LISA (Deffayet et al. 2007). Therefore, the DGP model does not necessarily produce significant deviations between gravitational and electromagnetic luminosity distances and it would, in fact, be difficult to distinguish from general relativity on the basis of the test illustrated in Fig. 1. On the other hand, since it is presently unclear whether infrared transparency and its consequences for GW propagation are generic to all cases of higher-dimensional gravity with large distance leakage (in particular when Lorentz symmetry is broken), Fig. 1 remains useful in providing a good measure of potential GW leakage.

5. Additional Signatures

Large distance leakage is only one of several possible modified gravity signatures in the GW signal from cosmologically-distant spacetime sirens. We discuss a few additional possibilities here, voluntarily adopting a simple phenomenological approach.

A first class of signatures resides in the GW polarization signal. In many modified gravity scenarios, additional polarizations exist beyond the two transverse quadrupolar ($+\times$) modes of general relativity (e.g., Will 2006). This is the case, for instance, in scalar-tensor theories (including the f(R) variety; Wands 1994; Maggiore & Nicolis 2000; Nakao et al. 2001) and vector-tensor theories (Bekenstein 2004; Jacobson & Mattingly 2004). Braneworld gravity scenarios are no exception since the Kaluza-Klein graviton extra polarizations can also in principle be radiated by various types of sources. The no-hair theorem is traditionally invoked to forbid any extra polarization modes in the signal from two coalescing black holes. However, it is well known that black hole unicity theorems fail to apply in the usual sense to higher dimensional theories (Emparan et al. 2002). Moreover, in DGP gravity the graviton extra polarization is expected to show up at large distances even for (possibly static) spherically symmetric space-times (Deffayet et al. 2002b). Black holes can also be hairy in theories of massive gravity (where the graviton carries extra polarizations with respect to those of a massless graviton), in particular when Lorentz invariance is broken (Blas et al. 2006, 2007; Dubovsky et al. 2007, and references therein).

A second class of signatures is related to the GW signal propagation velocity which, in modified gravity scenarios, can differ from the speed of light. Propagation can be subluminal (e.g., Dubovsky et al. 2005) or superluminal (e.g., Jacobson & Mattingly 2004). The possibility to time a GW signal propagated over cosmological distances, relative to the signal from a prompt electromagnetic counterpart causally associated with the black hole merger, may thus offer additional diagnostics of large-scale modified gravity. In addition, signatures may emerge because the number of spacetime dimensions available to gravity is odd. In this case, even a massless mode has a Green's function which does not vanish inside its light cone (see e.g., Cardoso et al. 2003). Consequences for GW propagation in braneworld gravity have been explored in part (Barvinsky & Solodukhin 2003), but not fully in the case when gravity is modified at large distances. Thus, the oddness of spacetime on cosmological scales, like in DGP gravity, could add to the set of modified gravity signatures.

Finally, a third class of signatures relates to the phase of the GW signal, which could deviate from general relativistic expectations once propagated over cosmological distances.

In summary, GWs from cosmologically-distant spacetime sirens may be valuable alternative probes of modified gravity since various signatures may exist in a GW signal propagated

over cosmological distances. For some of these signatures to become apparent, the identification of a host galaxy or an electromagnetic counterpart to the spacetime siren is required. Conversely, the absence of such signatures would constrain gravity modifications and provide a consistency check on other methods employed to discover the nature of dark energy.

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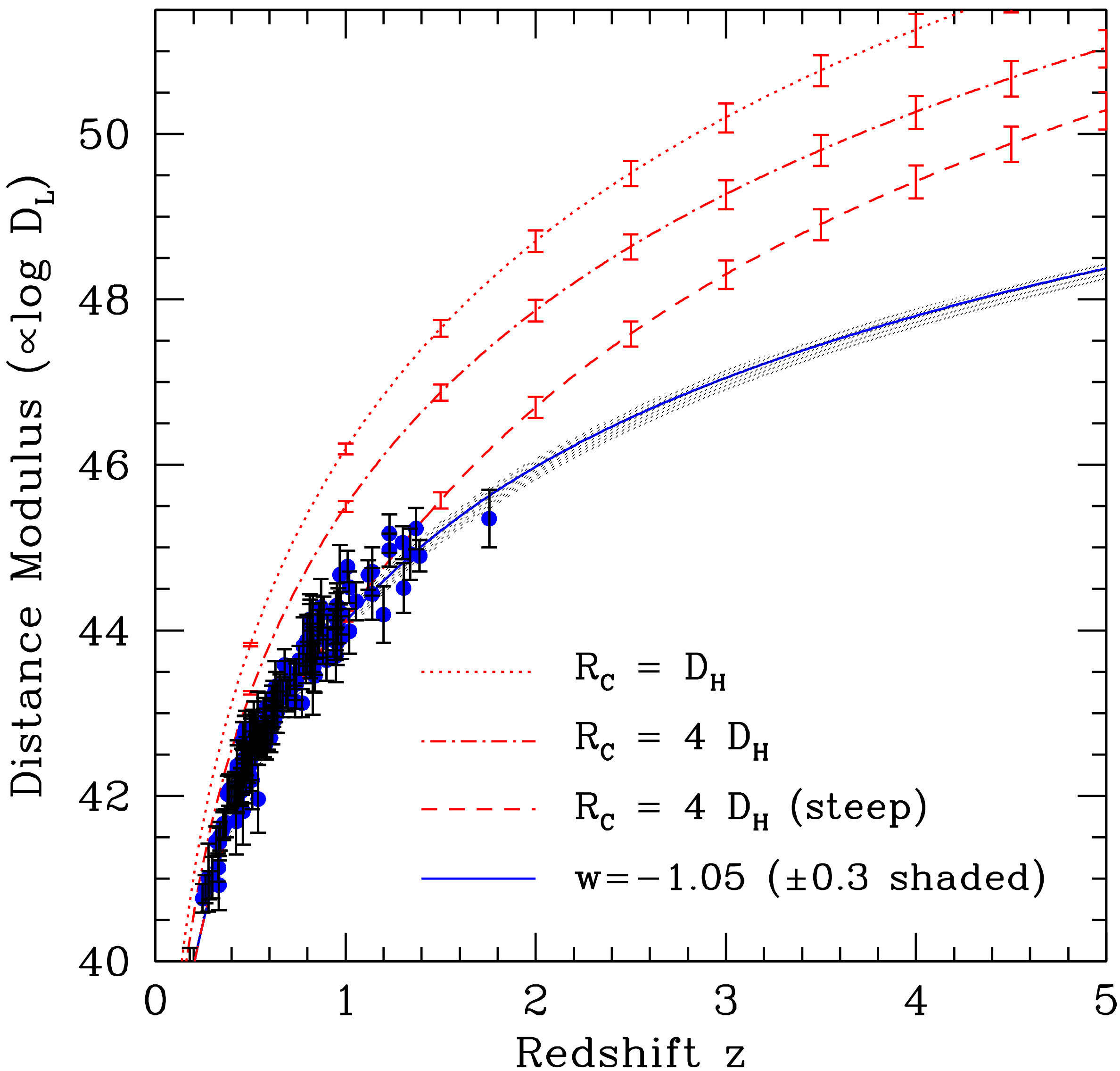

Fig. 1.— Electromagnetic Hubble diagram of a standard dark energy model (solid line + shaded region) and hypothetical gravitational Hubble diagrams for three modified gravity scenarios with biased gravitational luminosity distances from large distance gravitational leakage, beyond a cross-over scale R_c (top three lines).